Inverse Problem for Topological Photonics

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Introduction

Topological photonics has emerged as a frontier in controlling light propagation, with topological insulators playing a pivotal role in trapping light at specific frequencies. Traditionally, the design of these materials involves finding the right material composition to achieve the desired optical properties. Our method employs the transfer matrix method to solve the direct problem, which involves understanding how various topological parameters, such as those of materials, affect the frequencies at which light is trapped. By mapping out these relationships, our study then proceeded to tackle the inverse challenge: determining the combination of two distinct materials necessary to fabricate a topological insulator capable of confining light at a predetermined frequency.

Methods

Wave Propagation

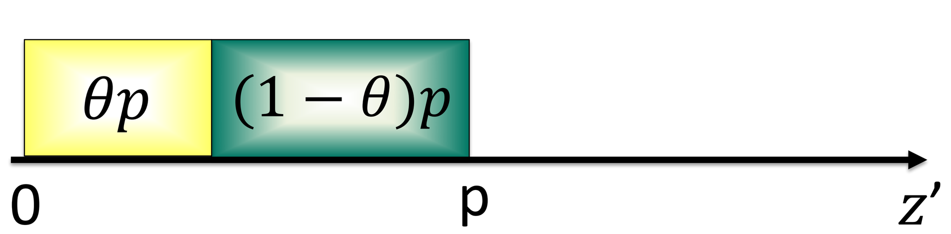
We used the Helmholtz Equation to determine the behavior of a wave in a periodically layered half-space. Since the medium is invariant in the x-direction, we assume the solution to be of the form $u(z)e^{i\xi x}$, where $u(z)$ satisfies

$$u(z)'' + \frac{\omega^2}{c(z)^2}u(z) - \xi^2 u(z) = 0$$

ω: *angular frequency*

c: speed of light

ξ: spatial wave number



Original Position z

To model the periodic medium, we assume. We normalize $z \to z'p$, $\omega \to \omega' p$, $\xi \to \xi' p$ (and drop the primes)

Propagation Matrix

The solution for the differential equation is:

Let $\sigma = \sqrt{\frac{\omega^2}{c^2} - \xi^2}$

$$u = A\cos(\sigma z) + B\sin(\sigma z)$$

$$u' = -\sigma A\sin(\sigma z) + \sigma B\cos(\sigma z)$$

$$\begin{bmatrix} u \\ u' \end{bmatrix}(0) = \begin{bmatrix} A \\ \sigma B \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ u' \end{bmatrix}(z) = \underbrace{\begin{bmatrix} \cos(\sigma z) & \frac{\sin(\sigma z)}{\sigma} \\ -\sigma\sin(\sigma z) & \cos(\sigma z) \end{bmatrix}}_{\text{Propagation Matrix}} \begin{bmatrix} u \\ u' \end{bmatrix}(0)$$

We will have different σ_A and σ_B for different materials A and B, θ is the fraction of each unit period with material A.

$$T_A = \begin{bmatrix} \cos(\sigma_A\theta) & \frac{\sin(\sigma_A\theta)}{\sigma_A} \\ -\sigma_A\sin(\sigma_A\theta) & \cos(\sigma_A\theta) \end{bmatrix}$$

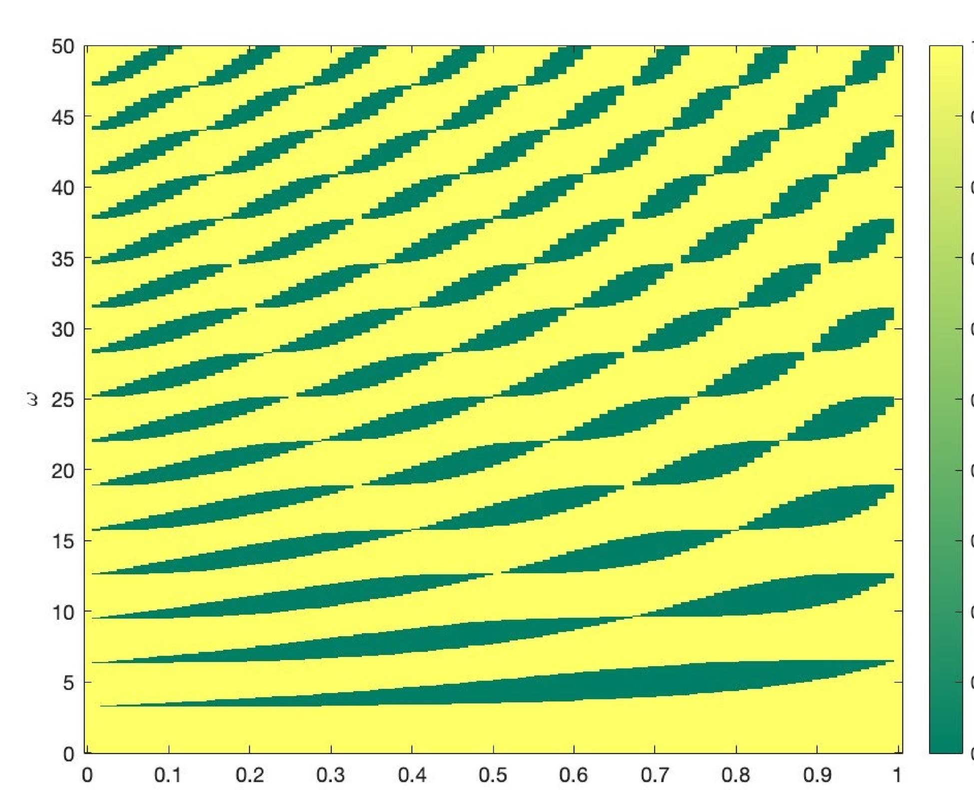
$$T_B = \begin{bmatrix} \cos(\sigma_B(1-\theta)) & \frac{\sin(\sigma_B(1-\theta))}{\sigma_B} \\ -\sigma_B\sin(\sigma_B(1-\theta)) & \cos(\sigma_B(1-\theta)) \end{bmatrix}$$

Transfer Matrix (In our model, we only consider one layer of A and one layer of B in a period, without refraction) Then the transfer matrix is defined as, with z' = np:

$$T^{(1)} = (T_A T_B)^n$$

Results (Direct Problem)

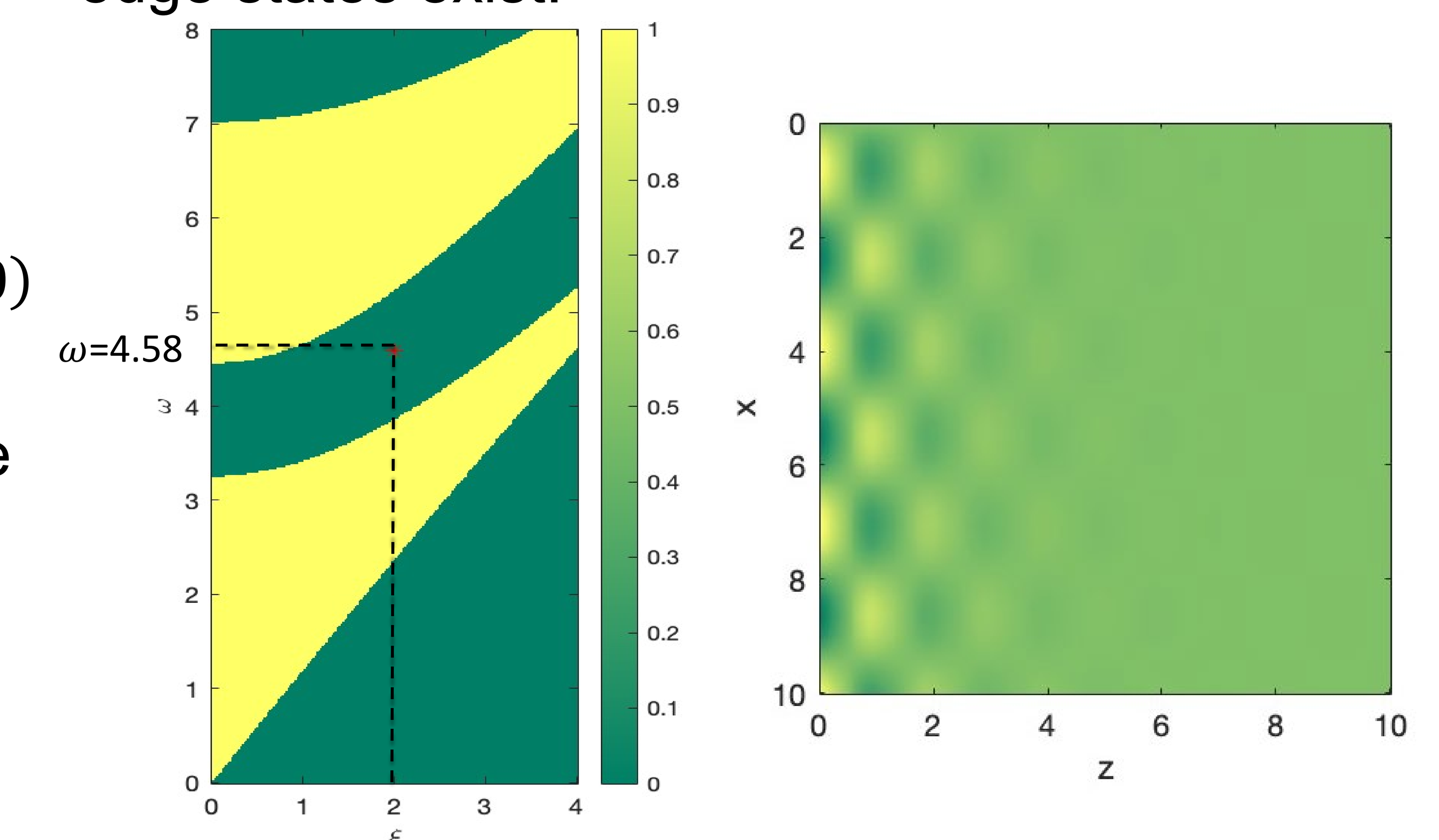
Edge States After calculating the transfer matrix, the edge states occur when the imaginary part of eigenvalues of $T^{(1)}$ is 0 (eigenvalues of $T^{(1)}$ are real).



Visualization of edge states with $\xi = 1, C_A = 2, C_B = 1$

Results (Edge Modes)

Identifying Edge States Let c_A and c_B be fixed. For any ξ and θ, there are frequencies for which edge states exist.



Visualization of wave propagation with $\xi = 2, \theta = 0.3$

Inverse Design Problem

Given a desired frequency ω_D and desired wave number ξ_D, find a structure that supports and edge mode at these parameters. That means finding c_A, c_B, θ that meets this design objective.

Future Steps

- Compute the direct problem efficiently using stated methods.
- Create datasets of edge modes at different parameter values.
- Develop a machine learning algorithm that solves the inverse design problem.

References

Pilozzi, Laura, et al. "Machine Learning Inverse Problem for Topological Photonics." Communications Physics, vol. 1, no. 57, 2018. https://doi.org/10.1038/s42005-018-0058-8.